Dynamic fatigue of alumina

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Alumina is used as lamp envelopes for high intensity discharge (HID) lamps that operate at temperatures near 1100 °C. These lamps are subjected to large temperature gradients and internal pressures that give rise to large stresses in the alumina lamp envelope. The stresses in commercial lamps are typically in the range of 40–80 MPa and must be sustained for upwards of 20 000 hrs. Knowing the fatigue and strength characteristics allows new designs to be screened before more detailed and time-consuming testing of actual lamps begins.

The alumina test samples consisted of tubes with a 3.0 mm outer diameter and a 1.1 mm inner diameter. The high purity (99.99+ wt%) samples were sintered above 1800 °C to high translucency and essentially full density. The alumina had an average grain intercept length of 12 microns, Fig. 1. A 3-point bend test fixture with an outer span of 16.76 mm was used in all of the tests. Crosshead speeds of 0.001, 0.00272, 0.01, 0.0272, 0.1, 0.31, 1, 2.717, and 10 mm/min. were used. These crossheads speeds resulted in stressing rates of 0.368, 1.0, 3.68, 10, 36.8, 114, 368, 1000, and 3681 MPa/s. Samples were tested at ambient conditions of 40–60% relative humidity and 20 °C. A limited number of tests were carried out between 1100 and 1500 °C. The fracture loads at each of the various stressing rates were measured and converted into maximum stress at the outer tensile surface using a finite element model.

A stressing rate of 3681 MPa/s was sufficient to suppress any subcritical crack growth because the values obtained at this stressing rate agreed well with limited in-house testing conducted in liquid nitrogen on the same material. The Weibull plot of the tests at 3681 MPa/s is shown in Fig. 2. The Weibull equation, as given in reference 1, is

$$\ln\left(\ln\left(\frac{1}{1-F}\right)\right) = m\ln\left(\frac{S}{S_o}\right) \quad (1)$$

where F is the failure probability, m is the Weibull modulus, S_o is the characteristic strength and S is the strength of the test piece. S_o and m are obtained by fitting the experimental strength data to Equation 1. The median fracture strengths and other Weibull parameters at each of the stressing rates are given in Table I. The median fracture strengths at each stressing rate were calculated for the Weibull parameters using a 0.5 failure probability. The strength of the alumina tested at a stressing rate of 362 MPa/s remained unchanged up to 1100 °C, Fig. 3. Because the strength is constant between 20 and 1100 °C and the high temperature testing of a large number of samples is problematic, the fatigue parameters measured at room temperature are assumed to be representative of temperatures up to 1100 °C.

The fracture data was processed by methods described in references 1 and 2 and the method will be only briefly outlined here. The dynamic fatigue experiments were fitted to

$$\sigma_f = \sigma_{fo}\left(\frac{\dot{\sigma}}{\dot{\sigma}_o}\right)^{\frac{1}{N+1}} \quad (2)$$

where σ_f is the fracture strength at the stressing rate $\dot{\sigma}$, $\dot{\sigma}_o$ is an arbitrarily chosen stressing rate, N is the slow crack growth parameter, and σ_{fo} is a constant obtained by fitting the experimental data.

In many situations proof testing is used to qualify a ceramic component for use. Proof testing is when a component is loaded to a specific load level (higher than the load the component will see in its application) and quickly unloaded. The load level is selected such that if the component can withstand the proof test load the part is guaranteed to survive the load in the application for the required time. The minimum time-to-fracture after proof testing is given by

$$t^{p}_{min} = \frac{\sigma_a^{-N}\sigma_p^{N-2}\sigma_{fo}^{N+1}}{\sigma_{IC}^{N-2}\dot{\sigma}_o(N+1)} \quad (3)$$

where σ_p is the proof test stress, σ_a is the applied stress (the stress the part must withstand in its application) and σ_{IC} is the fracture strength under inert conditions. Constants in Equation 3 can be grouped giving

$$t^{p}_{min} = B\sigma_a^{-N}\sigma_p^{N-2} \quad \text{where} \quad B = \frac{\sigma_{fo}^{N+1}}{\sigma_{IC}^{N-2}\dot{\sigma}_o(N+1)}. \quad (4)$$

For the case where proof testing is not used the minimum time-to-fracture is given by

$$\ln(t_f) = \frac{N-2}{m}\ln\left(\ln\left(\frac{1}{1-F}\right)\right) - N\ln(\sigma_a) + (N-2)\ln(\sigma_\theta) + \ln(B). \quad (5)$$

TABLE I Median strength values at each of the stressing rates. A stressing rate of 3681 MPa/s corresponds to the inert strength

Stressing rate (MPa/s)	Test temperature (°C)	Number of samples	Median strength (MPa)	Weibull modulus (m)	Characteristic strength (MPa)
0.368	20	10	203	10.2	210
1.0	20	10	192	10.8	199
3.68	20	20	210	10.6	218
10.0	20	15	212	14.0	218
36.8	20	46	218	9.2	228
114	20	20	225	14.7	230
368	20	20	246	8.7	258
1000	20	20	248	16.5	253
3681	20	86	356	8.8	371

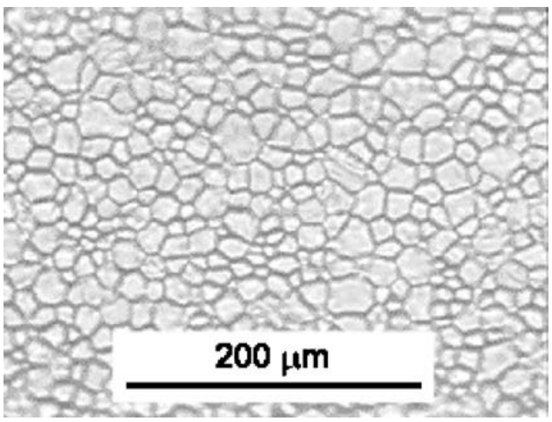

Figure 1 Surface grain structure of the alumina.

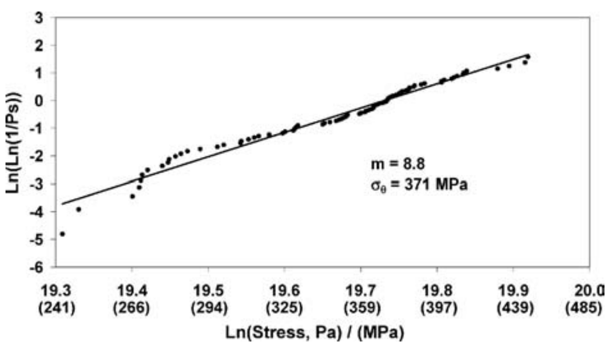

Figure 2 Weibull plot for tests run at a crosshead speed of 10 mm/min (3680 MPa/s). This stressing rate corresponds to the inert strength.

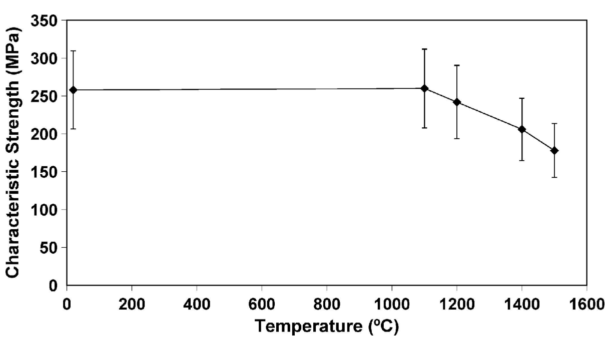

Figure 3 Plot of the fracture stress dependence on the test temperature. The error bars represent the 90% confidence interval.

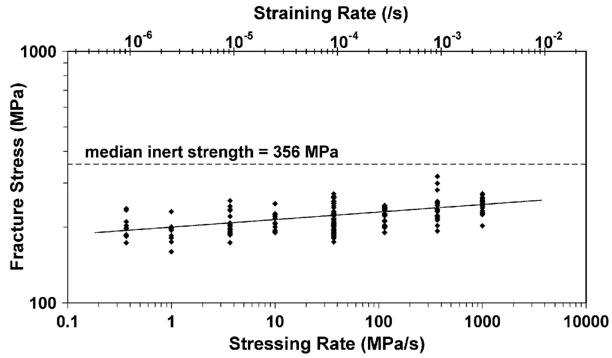

Figure 4 Plot of the fracture stress dependence on the stressing rate. Fitted using a linear regression analysis to Equation 2.

The variation in the lifetime given in Equation 5 was calculated using the law of propagating errors as

$$\begin{aligned}\operatorname{var}(\ln t_f) &= (N-2)^2 \operatorname{var}(\ln \sigma_\theta) + \operatorname{var}(\ln B) \\ &\quad + \left[\frac{N-2}{m^2}\ln\left(\ln\left(\frac{1}{1-F}\right)\right)\right]^2 \operatorname{var}(m) \\ &\quad + \left[\frac{\ln\left(\ln\left(\frac{1}{1-F}\right)\right)}{m} - \ln\sigma_a + \ln(\sigma_\theta)\right]^2 \operatorname{var}(N) \qquad (6)\end{aligned}$$

where the variation of $\ln(\sigma_\theta)$, $\ln(B)$, m and N are 0.0193, 1.21×10^{-4}, 0.0278 and 48.7, respectively.

Fig. 4 shows the plot of Equation 2 and Table II lists the derived fracture parameters. The fracture parameters listed in Table II were used in Equations 3–5 to generate Figs 5 and 6.

Fig. 5 is called a design diagram and can be used to determine the proof test stress required to guarantee a desired life. There are 2 sets of lines on this diagram. The lines labeled 2.7, 2.9 and 3.1 are different proof test ratios. The lines labeled 10^{-4}, 10^{-3}, 10^{-2} and 10^{-1} are for the stated failure probabilities without proof testing.

TABLE II Calculated fracture parameters

Parameter	Value
Inert characteristic strength (σ_θ)	371 MPa
Inert median strength (σ_o)	356 MPa
Inert weibull modulus (m)	8.8
Reference stressing rate ($\dot{\sigma}_o$)	36.81/s
Reference fracture stress (σ_{fo})	223 MPa
N	32.4
B	5.97×10^{-3} MPa^2-s

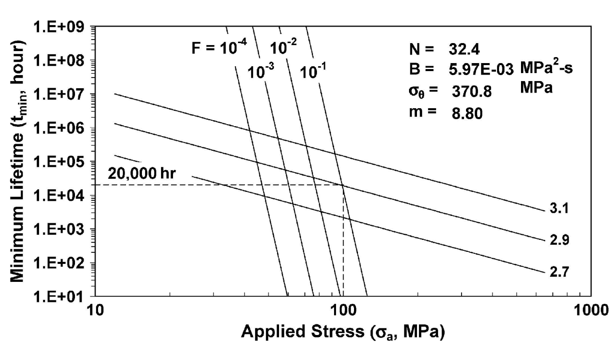

Figure 5 Design diagram calculated based on values given in Table II.

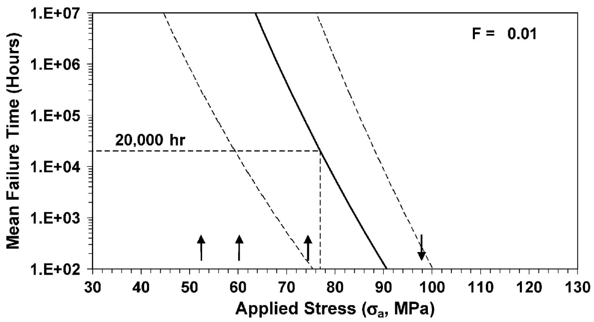

Figure 6 Plot of the minimum lifetime for a 1% failure probability with the 90% confidence limits displayed. The up arrows indicate stress levels in production parts with a life of 20 000 hrs. The down arrow indicates the stress level in test parts with known reliability problems.

The dotted line in Fig. 5 serves as an example. If we desire a part to withstand a 100 MPa stress level for 20 000 hrs then we need to subject all of the parts to a proof test at 290 MPa (2.9 × 100 MPa). If we do not do a proof test then the parts will have approximately a 10% chance of failing before the required 20 000 hrs time. Fig. 6 shows the diagram for the case without proof testing (Equation 5) and with 90% confidence interval included. The uncertainty in the exponent N was the largest contributor to the uncertainty in the lifetime. For a 1% failure rate and a minimum lifetime of 20 000 hrs the stress must be less than 77 MPa, the solid line in Fig. 6. It is informative to compare this level of 77 MPa to the stress for a 1% failure probability based on fast fracture calculated with Equation 1, which is 220 MPa.

The maximum allowable stress level calculated by the fast fracture strength is significantly higher than that based on the fatigue results. Basing a design on the fast fracture results would result in many early failures.

The arrows in Fig. 6 indicate the stress levels in current production parts. These arrows indicate that the mean line provides a good estimate of the part lifetimes and that the lower 90% confidence limit is over conservative.

References

1. J. E. RITTER, JR., "Engineering Design and Fatigue Failure of Brittle Materials", in Fracture Mechanics of Ceramics, v4 edited by R. C. Brandt, D. P. H. Hasselman and F. F. Lange (Plenum Press, 1978) p. 667.
2. D. F. JACOBS and J. E. RITTER, Jr., *J. Amer. Ceram. Soc.* **59**(11/12) (1976) 481.

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